Periodically Loaded Nonreciprocal Transmission Lines for Phase-Shifter Applications

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Abstract—**Nonreciprocal transmission lines periodically loaded with thin metallic diaphragms are analyzed using the wave transmission matrix approach. An approximate equivalent circuit representation of the diaphragm is proposed and discussed. Using this representation, the differential phase-shift and impedance characteristics of the periodically loaded line are computed for assumed parameters, for "shunt-capacitance" and "shunt-industance" loading. The range of validity of the approximate results is examined using a certain criterion. The differential phase shift for both capacitive and inductive loading is found to be greater than that of the unloaded line and the results show the same general trends as those previously observed experimentally.**

I. INTRODUCTION

RECENT experimental evidence [1], [2] indicates that periodic loading of a version of the twin-slab waveguide ferrite phase shifter [3] by disks, apertures, or a combination of both, yields interesting and useful differential phase-shift characteristics. Disk or aperture loading was found to substantially increase the differential phase shift per unit length of the device, while alternate disk and aperture loading showed that it was possible to obtain a device with a flat response over a wide frequency range. The analysis presented in this paper has been carried out in an attempt to explain some of these experimental observations and to provide some understanding of the effect of periodic loading of nonreciprocal transmission lines.

The approach adopted is essentially a modification of the wave transmission matrix analysis of periodic reciprocal structures. It takes into account the "external characteristics" of the nonreciprocal unloaded lines (phase coefficients and characteristic impedances), but does not depend on the knowledge of their exact geometry or on how nonreciprocal propagation is achieved. Certain assumptions and approximations are made. Subject to these limitations, solutions are obtained for the phase coefficients and the Bloch wave impedances of the periodically loaded lines. Numerical values are given for a wide range of assumed parameters.

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II. THE NONRECIPROCAL LINE PERIODICALLY LOADED WITH THIN DIAPHRAGMS

Consider a length L of a "smooth" lossless infinite nonreciprocal line, as shown in Fig. 1(a). Using the wave transmission matrix representation [4], the amplitudes of the forward- and the backward-traveling waves, g and h, at the input and output planes i–i and o–o can be related in the following manner:

$$\begin{bmatrix} g_i \\ h_i \end{bmatrix} = \begin{bmatrix} \exp(j\beta^+ L) & 0 \\ 0 & \exp(-j\beta^- L) \end{bmatrix} \begin{bmatrix} g_o \\ h_o \end{bmatrix} \tag{1}$$

where β^+ and β^- are the phase coefficients of the line for propagation in the forward (+) and backward (−) directions, respectively.

The line is now periodically loaded, interval L, with thin diaphragms. In a reciprocal line, these diaphragms are represented in an equivalent circuit by shunt susceptances. In this case, due to the nonreciprocal properties

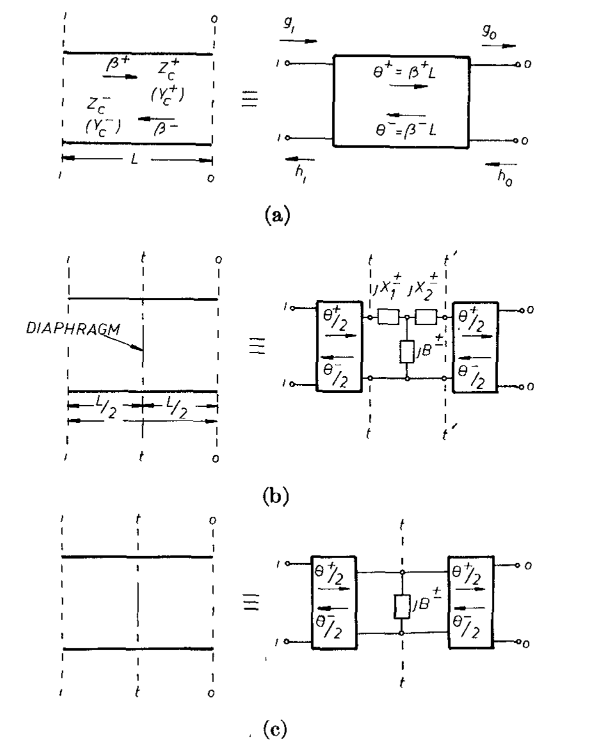

Fig. 1. Circuit representation of nonreciprocal transmission line section. (a) Unloaded line. (b) Proposed exact equivalent circuit (c) Proposed approximate equivalent circuit.

of the line, a nonsymmetrical three-element T network would provide a more general equivalent circuit of the diaphragm, as shown in Fig. 1(b), the elements having different values for the opposite directions of propagation. As a first approximation, however, it will be assumed that the diaphragm can be represented by a single shunt susceptance (of different magnitudes for the two directions of propagation), as shown in Fig. 1(c). This assumption should not be unreasonable if the differences between the properties of the nonreciprocal line in the two directions of propagation are not too great.

Provided the coupling between successive diaphragms can be neglected, the amplitudes of the forward- and backward-traveling waves at the input and output of the loaded line section are related by

$$\begin{bmatrix} g_i \\ h_i \end{bmatrix} = \frac{1}{T^+} \begin{bmatrix} \exp\left(\frac{j\theta^+}{2}\right) & 0 \\ 0 & \exp\left(\frac{-j\theta^-}{2}\right) \end{bmatrix} \begin{bmatrix} 1 & -R^- \\ R^+ & T^+T^- - R^+R^- \end{bmatrix} \begin{bmatrix} \exp\left(j\frac{\theta^+}{2}\right) & 0 \\ 0 & \exp\left(-j\frac{\theta^-}{2}\right) \end{bmatrix} \begin{bmatrix} g_o \\ h_o \end{bmatrix}$$

$$= \frac{1}{T^+} \begin{bmatrix} \exp(j\theta^+) & -R^- \exp[j\tfrac{1}{2}(\theta^+ - \theta^-)] \\ R^+ \exp[j\tfrac{1}{2}(\theta^+ - \theta^-)] & (T^+T^- - R^+R^-)\exp(-j\theta^-) \end{bmatrix} \begin{bmatrix} g_o \\ h_o \end{bmatrix} \tag{2}$$

where

$$T^{\pm} = \frac{2}{2 + j\bar{B}_e^{\pm}}$$

$$R^{\pm} = \frac{-j\bar{B}_e^{\pm}}{2 + j\bar{B}_e^{\pm}}. \tag{3}$$

$T^{\pm}$ and $R^{\pm}$ are voltage transmission and reflection coefficients, respectively. $\bar{B}_e^+$ and $\bar{B}_e^-$ are (approximate) "effective" normalized shunt susceptances of the diaphragm for propagation in the forward and backward directions, respectively. Relations (3) are derived in the Appendix.

It is important to note at this point that the determinant in (2) does not generally satisfy the conditions required for a physical nonreciprocal loss-free two-port. This is, of course, a direct result of the equivalent circuit approximation where the diaphragm is represented by a single shunt susceptance in each direction. The extent of usefulness of this approximation is investigated numerically in Section V.

Equation (2) may be written in the form:[1]

$$\begin{bmatrix} g_i^+ \\ h_i^+ \end{bmatrix} = \begin{bmatrix} A_{11}^+ & A_{12}^+ \\ A_{21}^+ & A_{22}^+ \end{bmatrix} \begin{bmatrix} g_o^+ \\ h_o^+ \end{bmatrix} \tag{2a}$$

where

$$A_{11}^+ = \frac{1}{T^+}\exp(j\theta^+)$$

$$A_{12}^+ = \frac{-R^-}{T^+}\exp[j\tfrac{1}{2}(\theta^+ - \theta^-)]$$

$$A_{21}^+ = \frac{R^+}{T^+}\exp[j\tfrac{1}{2}(\theta^+ - \theta^-)]$$

$$A_{22}^+ = \frac{T^+T^- - R^+R^-}{T^+}\exp(-j\theta^-). \tag{4}$$

If a Bloch wave is to propagate in the forward direction in the infinite structure of identical loaded sections in cascade, the propagation constant per section, $\gamma^+ = \alpha^+ + j\phi^+$, is a solution of the equation below [4]:

$$\begin{vmatrix} A_{11}^+ - e^{\gamma} & A_{12}^+ \\ A_{21}^+ & A_{22}^+ - e^{\gamma} \end{vmatrix} = 0. \tag{5}$$

The second solution, $\gamma^- = \alpha^- + j\phi^-$, is for propagation in the negative direction. In general, one may write the eigenvalue equation in the form

$$\begin{vmatrix} A_{11}^{\pm} - e^{\gamma} & A_{12}^{\pm} \\ A_{21}^{\pm} & A_{22}^{\pm} - e^{\gamma} \end{vmatrix} = 0. \tag{5a}$$

III. THE EQUIVALENT SHUNT SUSCEPTANCES

The shunt susceptances of the diaphragms will naturally depend on the geometry of the line and on the type, shape, and dimensions of the diaphragm as well as on the frequency and propagating mode. The discussion presented in this section is intended primarily for the purpose of obtaining an appreciation of the effect of diaphragms in nonreciprocal lines, which are also generally inhomogeneous.

Consideration of the reciprocal case may provide some guidance: it is possible to derive analytic expressions [4], [5], or use field matching computer methods [6] to com-

[1] If the input and output terminals are interchanged, one obtains

$$\begin{bmatrix} g_i^- \\ h_i^- \end{bmatrix} = \begin{bmatrix} A_{11}^- & A_{12}^- \\ A_{21}^- & A_{22}^- \end{bmatrix} \begin{bmatrix} g_o^- \\ h_o^- \end{bmatrix} \tag{2b}$$

where A_{11}^-, $A_{12}^- \cdots$ are appropriately defined in accordance with (4).

pute accurately the normalized shunt susceptance (or reactance) of a particular diaphragm in a specific homogeneous line. The method in [6] has been extended to deal with transverse discontinuities in inhomogeneous waveguides [7]. Experimental confirmation is also provided in [7]. Further study of the latter case revealed a strong dependence of the normalized shunt susceptance of a transverse discontinuity in an inhomogeneous waveguide on the propagation coefficient β of the dominant propagating mode. This may be illustrated by an example. Consider the configuration shown in Fig. 2. The frequency and the dimensions of the waveguide and the inductive window were kept constant, while the phase coefficient β was varied by changing the thickness t and the permittivity ϵ_r of the dielectric layer. The normalized shunt susceptance $\bar{B}$ of the inductive window was calculated using the method in [7]. Normalization was done with respect to the characteristic admittance of the inhomogeneous waveguide in each case. The quantity $\bar{B}/\bar{B}_{(t=0)}$ is plotted as a function of $\beta/\beta_{(t=0)}$ in Fig. 3(a). In the region considered, it is seen that $\bar{B}$ is quite sensitive to the value of β. Moreover, the variation of $\bar{B}$ as a function of β for any particular thickness of the dielectric is both smooth and monotonic. As t/λ_0 (λ_0 is the free-space wavelength) decreases, $\bar{B}$ seems to depend almost exclusively on the value of β, irrespective of the values of the thickness or the permittivity of the dielectric layer. This is depicted in Fig. 3(b) where the scales have been expanded to show this behavior adequately. It is not, however, argued that the behavior just illustrated would hold to this degree in all cases of transverse discontinuities and inhomogeneous waveguides. The example merely serves to indicate that the normalized shunt reactance of the discontinuity in the inhomogeneous waveguide is strongly dependent on the phase coefficient.

Now let us examine, in the light of this evidence, a similar configuration of a nonreciprocal waveguide with phase coefficients β^+ and β^-. If the difference between β^+ and β^- is not too great, a transverse discontinuity may be approximately represented by a single shunt element; the susceptance of this element would assume two different values $B^{\pm}$ for the two directions of propagation. When normalized to the appropriate characteristic admittance of the line, Y_c^+ or Y_c^-, the resultant values $\bar{B}^{\pm}$ are expected to be different, as it is argued that these values would be dependent mainly on the values of $\beta^{\pm}$. Thus one could suggest a method for estimating the values of $\bar{B}^{\pm}$

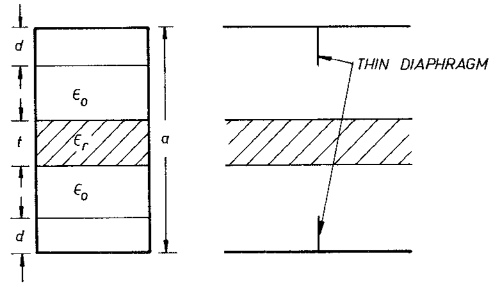

Fig. 2. Inhomogeneously filled rectangular waveguide with inductive window. $a/\lambda_0 = 0.7473$; $d/\lambda_0 = 0.1$.

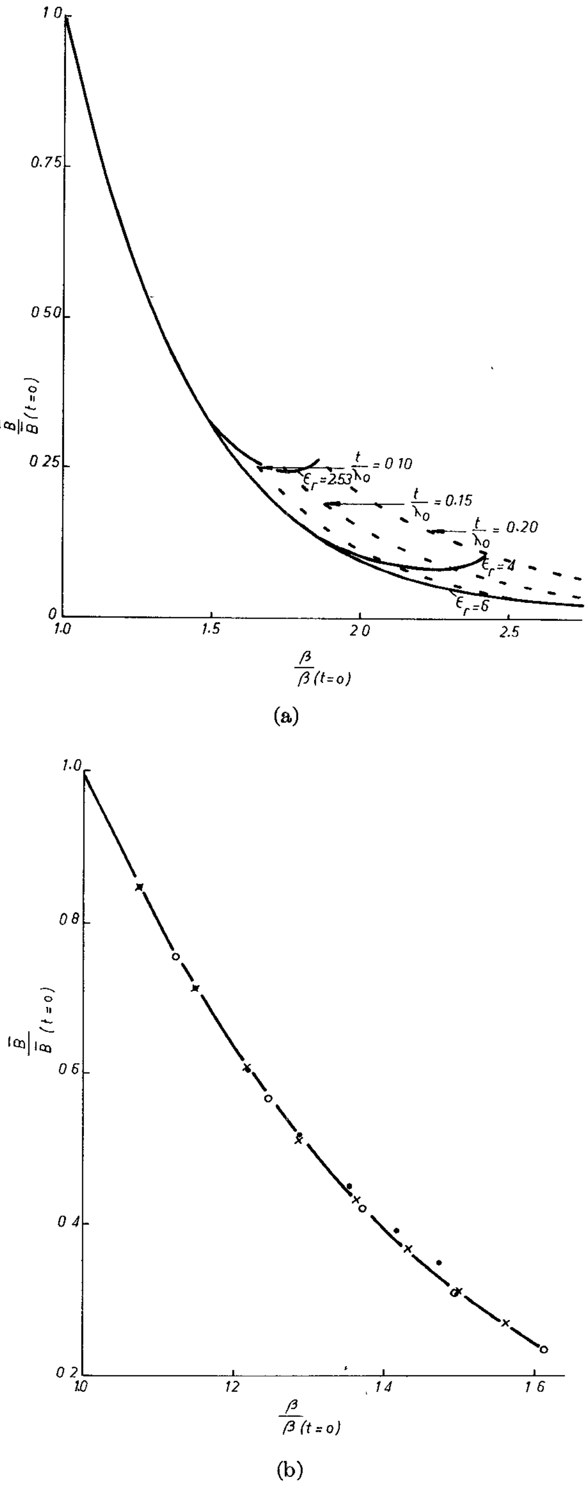

Fig. 3. Variation of normalized susceptance of the inductive window of Fig. 2 as a function of phase coefficient. (a) For relatively large values of t/λ_0. (b) For small values of t/λ_0: ● $\epsilon_r = 2.53$, t/λ_0 varies in steps of 0.01; × $\epsilon_r = 4.00$, t/λ_0 varies in steps of 0.005; ○ $\epsilon_r = 6.00$, t/λ varies in steps of 0.005.

of the approximate equivalent circuit of Fig. 1(b). This would involve computing $\bar{B}$ for two similar configurations using dielectrics whose permittivities yield phase coefficients equal to β^+ and β^- of the nonreciprocal line. This method might be satisfactory only in limited cases and, in general, there is a need to determine the parameters of the exact equivalent circuit of Fig. 1(b). Experimental work is presently planned to attempt just this.[2]

IV. IMPEDANCE

The characteristic (Bloch wave) impedances of a periodically loaded line, $Z_B^\pm$, when the line is properly terminated at specific planes, are not unique, but depend on the location of the terminal planes chosen. Let the input and output terminal planes be symmetrically situated with respect to the loading diaphragm, as shown in Fig. 1(c), and let us connect an impedance Z_B^+ at terminals o–o (or Z_B^- at terminals i–i, depending on direction of propagation) of the unit cell. Z_B^+ is then equal to the input impedance of the cell, Z_{i-i}, when the cell is terminated in Z_B^+, and is thus evaluated by equating Z_B^+ to the ratio of the total voltage to the total current at the terminals i–i. By applying relation (2), we have for the forward- and backward-traveling voltage waves, g and h:

$$g_i^\pm = (A_{11}^\pm + \rho^\pm A_{12}^\pm) g_o^\pm$$

and

$$h_i^\pm = (A_{21}^\pm + \rho^\pm A_{22}^\pm) g_o^\pm$$

where

$$\rho^\pm = \frac{h_o^\pm}{g_o^\pm} = \frac{Z_B^\pm - Z_c^\pm}{y^\pm Z_B^\pm + Z_c^\pm}$$

are "characteristic" voltage reflection coefficients at the load terminals.

$$Z_B^\pm = Z_{i-i}^\pm = \frac{A_{11}^\pm + \rho^\pm A_{12} + A_{21}^\pm + \rho^\pm A_{22}^\pm}{Y_c^\pm(A_{11}^\pm + \rho^\pm A_{12}^\pm) - Y_c^\mp(A_{21}^\pm + \rho^\pm A_{22}^\pm)}$$

where $Y_c^\pm = 1/Z_c^\pm$ are the characteristic admittances of the unloaded nonreciprocal line in the two directions of propagation. Normalized to the characteristic impedances of the unloaded line, $Z_c^\pm$,

$$\bar{Z}_B^\pm = \frac{A_{11}^\pm + \rho^\pm A_{12} + A_{21}^\pm + \rho^\pm A_{22}}{(A_{11}^\pm + \rho^\pm A_{12}) - y^\pm(A_{21}^\pm + \rho^\pm A_{22}^\pm)}$$

$$y^\pm = \frac{Y_c^\mp}{Y_c^\pm}. \quad (6)$$

V. NUMERICAL RESULTS

The differential phase-shift and impedance characteristics of the periodically loaded line have been computed for a wide range of assumed parameters. Before presenting these results, it is thought useful to examine the effect of the equivalent circuit approximation used. For the reason mentioned in Section II, the propagation constants per cell, calculated using the single shunt element approximation, are complex; $\gamma^\pm = \alpha^\pm + j\phi^\pm$. The ratio $\alpha^\pm/\phi^\pm$ can be either positive or negative depending on the direction of propagation and the type of loading. Since the theoretical $\alpha^\pm$ should be indentically equal to zero in the pass range when using the exact equivalent circuit (no dissipation is assumed in the system), the value $|\alpha^\pm/\phi^\pm|$ could be taken as an indication of the degree to which the equivalent representation of Fig. 1(c) is useful. A typical behavior of $|\alpha^\pm/\phi^\pm|$ for shunt-capacitance loading is shown in Fig. 4.

A. Shunt-Susceptance Loading

Fig. 5 shows a plot of the differential phase shift $\Delta\phi = \phi^+ - \phi^-$ per section of a shunt-capacitance loaded line as a function of $\bar{B}_e$, for various values of a parameter $k =$

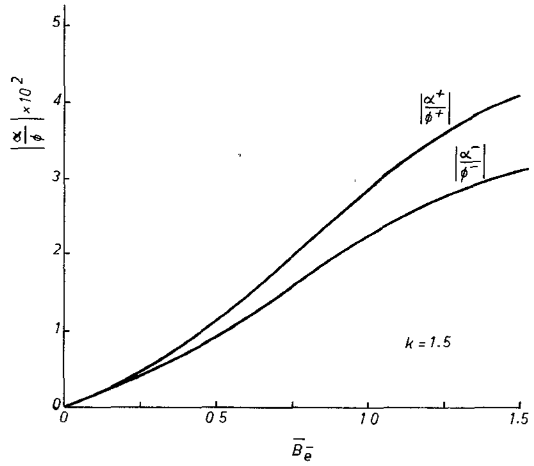

Fig. 4. Relative magnitudes of "attenuation" to phase constants of a loaded line section.

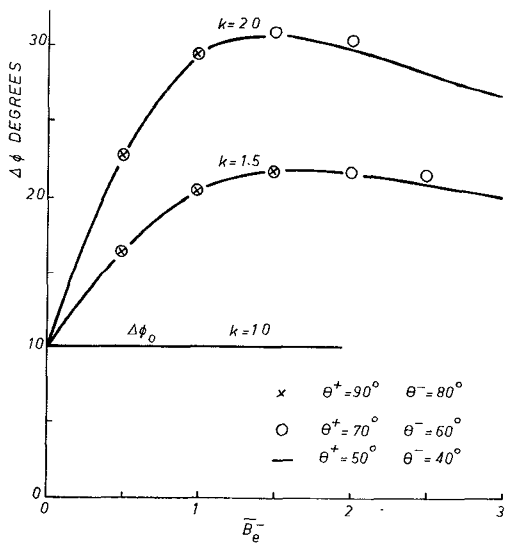

Fig. 5. Differential phase-shift characteristics of "shunt-capacitance" loaded line.

[2] Current Ph.D. dissertation work by W. K. McRitchie, University of British Columbia, Vancouver, B. C., Canada.

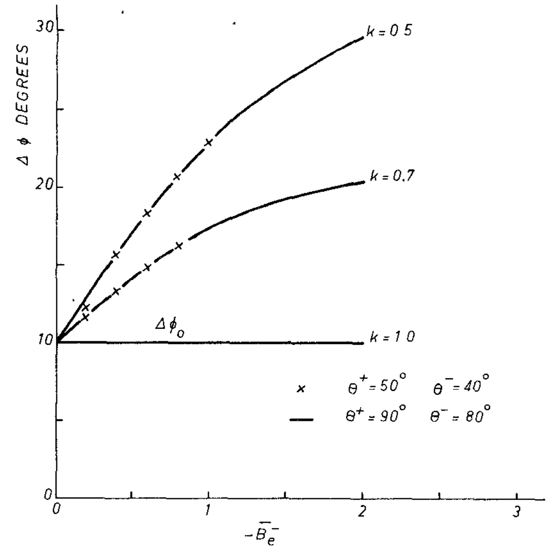

Fig. 6. Differential phase-shift characteristics of "shunt-inductance" loaded line.

$\bar{B}_e^+/\bar{B}_e^-$ and with $\theta^+ = 50°$ and $\theta^- = 40°$. $\Delta\phi$ values, for other values of $\theta^\pm$, $(\theta^+ - \theta^- = 10°)$, are indicated by the crosses and the circles on the same diagram. It is noticed that these lie very close to or on the $\theta^+ = 50°$ curve, suggesting that, for the same value of the loading susceptance, $\Delta\phi$ is fairly independent of the loading interval. $\Delta\phi$ is, however, very sensitive to the value of k, and, as expected, cutoff occurs at lower values of $\bar{B}_e$ for the higher values of θ. For shunt-inductance loading, the corresponding information is plotted in Fig. 6. It is interesting to note that $\Delta\phi > \Delta\phi_o$ for both shunt-capacitance and shunt-inductance loading. $\Delta\phi_o$ is the differential phase shift of the unloaded line.

B. Impedance

The normalized Bloch wave impedances as given by (6) were computed for different values of the parameter k lying in the range 1.0–2.0 in the case of capacitive loading (and 1.0–0.5 for inductive loading), and for different values of y^- in the range 1.0–2.0. It was found that while the impedance was strongly dependent on k, it was not very sensitive to y within the ranges considered.

Typical plots of the real and imaginary parts of $\bar{Z}_B^\pm$ as a function of $\bar{B}_e^-$ are shown in Figs. 7 and 8 for capacitive and inductive loading, respectively. The imaginary part of the impedance is relatively small, but increases sharply when the real part becomes very small in the capacitive case. This is again a result of the equivalent circuit approximation which appears to break down for heavy values of loading and near the cutoff region of the periodic structure.

VI. CONCLUSIONS

The approach adopted is quite general and depends only indirectly on the type of unloaded nonreciprocal transmission line. It requires knowledge of the equivalent circuit parameters of the loading diaphragm. The analysis

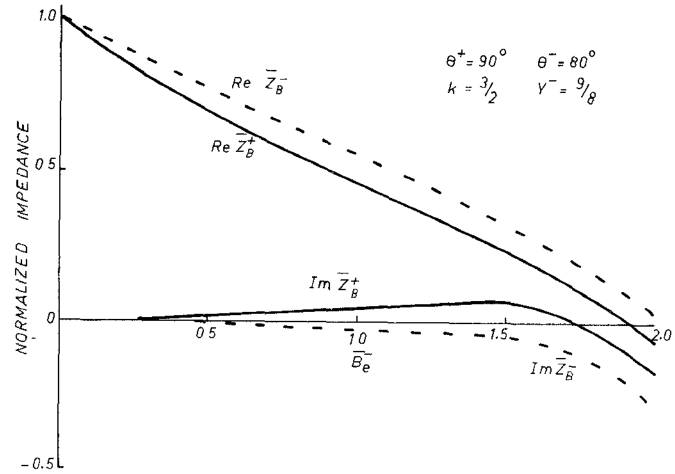

Fig. 7. Normalized characteristic impedance of "shunt-capacitance" loaded line.

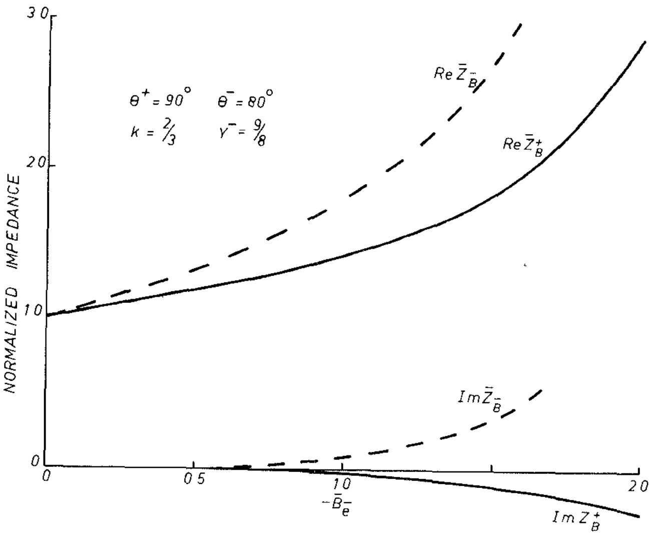

Fig. 8. Normalized characteristic impedance of "shunt-inductance" loaded line.

is approximate, but some measure of the effect of approximations is provided. The approximations used are essentially contained in the equivalent circuit representation of the loading diaphragms, and result in considerable simplification of the computations. In order to obtain quantitative (design) values in specific cases, it is necessary to be able to determine accurately the parameters of the exact equivalent circuit of the loading diaphragm in the inhomogeneous nonreciprocal waveguide under consideration. Most probably this would be feasible only experimentally.

Although the computed characteristics are based on assumed values, the results are meaningful and, where applicable, they show the same trends as existing experimental evidence.

Based on the analysis presented in the paper, the following points are of special interest concerning the characteristics of the periodically loaded nonreciprocal lines.

1) The differential phase-shift characteristics appear to be virtually independent of the loading interval within

the pass range and for the same value of the loading susceptance. Cutoff, however, strongly depends on the loading interval, as would be expected.

2) Both inductive and capacitive shunt loading result in an *increase* in the differential phase shift of the unloaded line (as observed experimentally by Spaulding [1]).

3) The additional differential phase shift (over that of the unloaded line) is very sensitive to the parameter k, which in turn is a function of the phase coefficients and impedances of the lines.

It should be emphasized that the above characteristics, particularly 1) and 2), were obtained by assuming "pure" shunt loading. Consideration of the exact equivalent circuit of the loading diaphragm, where series elements are present, would undoubtedly result in modified characteristics.

APPENDIX

DERIVATION OF $R^\pm$ AND $T^\pm$

Referring to Fig. 9, the total voltage V_t and total current I_t at the terminals t–t (at $z = 0$) for a wave propagating in the positive direction of z on a nonreciprocal transmission line are given by

$$V_t = V^+ + V^-$$

and

$$I_t = I^+ + I^- = Y_c^+V^+ - Y_c^-V^-$$

the total admittance at terminals t–t

$$Y_t = \frac{I_t}{V_t} = \frac{Y_c^+ - Y_c^-R^+}{1 + R^+}, \qquad R^+ = \frac{V^-}{V^+}$$

$$= Y_c^+ + jB^+.$$

Rearranging, we obtain

$$R^+ = \frac{-jB^+}{Y_c^+ + Y_c^- + jB^+}.$$

Similarly, we can obtain

$$R^- = \frac{-jB^-}{Y_c^+ + Y_c^- + jB^-}.$$

If we normalize $B^\pm$ to the characteristic admittances $Y_c^\pm$, $\bar{B}^\pm = B^\pm/Y_c^\pm$, and define $y^\pm = Y_c^\mp/Y_c^\pm$, then we can write

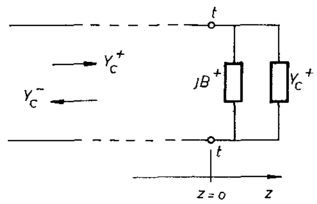

Fig. 9. Terminated nonreciprocal line for a wave incident in the positive direction of z.

$$R^\pm = \frac{-j\bar{B}^\pm}{1 + y^\pm + j\bar{B}^\pm}.$$

The above expression can be simplified further by defining "effective" normalized shunt susceptances

$$\bar{B}_e^\pm = \frac{2\bar{B}^\pm}{1 + y^\pm}.$$

Thus the expression for $R^\pm$ reduces to

$$R^\pm = \frac{-j\bar{B}_e^\pm}{2 + j\bar{B}_e^\pm}$$

giving

$$T^\pm = 1 + R^\pm = \frac{2}{2 + j\bar{B}_e^\pm}.$$

These final expressions for $R^\pm$ and $T^\pm$ are similar to those obtained in the case of reciprocal transmission lines.

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